

# A Constraint on the Distance Scale to Cosmological Gamma-Ray Bursts

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## ABSTRACT

If  $\gamma$ -ray bursts are cosmological in origin, the sources are expected to trace the large-scale structure of luminous matter in the universe. I use a new likelihood method that compares the counts-in-cells distribution of  $\gamma$ -ray bursts in the BATSE 3B catalog with that expected from the known large-scale structure of the universe, in order to place a constraint on the distance scale to cosmological bursts. I find, at the 95% confidence level, that the comoving distance to the “edge” of the burst distribution is greater than  $630 h^{-1}$  Mpc ( $z > 0.25$ ), and that the nearest burst is farther than  $40 h^{-1}$  Mpc. The median distance to the nearest burst is  $170 h^{-1}$  Mpc, implying that the total energy released in  $\gamma$ -rays during a burst event is of order  $3 \times 10^{51} h^{-2}$  ergs. None of the bursts that have been observed by BATSE are in nearby galaxies, nor is a signature from the Coma cluster or the “Great Wall” likely to be seen in the data at present.

*Subject headings:* gamma rays: bursts — large-scale structure of universe — methods: statistical

## 1. Introduction

The origin of  $\gamma$ -ray bursts is still unknown and is currently the subject of a “great debate” in the astronomical community. Do the bursts have a Galactic origin (Lamb 1995) or are they cosmological (Paczynski 1995)? And what is their distance scale?

In this Letter, I do not attempt to answer the first question, but rather, I show that *if* one assumes that  $\gamma$ -ray bursts are cosmological in origin, one can begin to answer the second question and place a constraint on the distance scale to the bursts. This is because cosmological bursts are expected to trace the large-scale structure of luminous matter in the universe (Lamb & Quashnock 1993, hereafter, LQ). The constraint comes from comparing the *expected* clustering pattern of bursts on the sky — which will depend on their distance scale because of projection effects — with that *actually observed*. The observed angular distribution is in fact quite isotropic (Briggs et al. 1996); hence, only a lower limit to the distance scale can be placed because a sufficiently large distance will always lead to a sufficiently isotropic distribution on the sky.

Hartmann and Blumenthal (1989) first used the absence of a significant angular correlation function in a small sample of  $\gamma$ -ray bursts to put a lower limit of  $71 h^{-1}$  Mpc on the distance scale to the bursts.<sup>2</sup> This lower limit was subsequently improved to  $150 h^{-1}$  Mpc (Blumenthal, Hartmann, & Linder 1994) using the larger BATSE 1B catalog (Fishman et al. 1994).

Here I use a powerful new likelihood method, which I had previously developed to analyze repeating of  $\gamma$ -ray bursts in the BATSE 1B and 2B catalogs (Quashnock 1995), to compare the observed counts-in-cells distribution in the new BATSE 3B catalog (Meegan

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<sup>2</sup>I follow the usual convention and take  $h$  to be the Hubble constant in units of  $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$ .

et al. 1996) with that expected for bursts at cosmological distances. I describe this method and calculate the expected counts-in-cells distribution in § 2.1, using the angular correlation function computed in § 2.2. I present my results on the burst distance scale in § 3, and discuss some implications of this work in § 4. This work was presented in preliminary form elsewhere (Quashnock 1996).

## 2. Likelihood Method

### 2.1. Counts-in-Cells Distribution

Let  $N_{\text{cell}}$  be a large number of circular cells, each centered on a random position on the sky. Each cell is of fixed solid-angle size  $\Omega = 2\pi(1 - \cos \theta_{\text{rad}})$ , where  $\theta_{\text{rad}}$  is the angular radius of the cell. I set the number of cells to be such that any part of the sky is covered, on average, by one cell; hence,  $N_{\text{cell}} = 4\pi/\Omega$ . Let  $C_N$  to be the number of these cells having  $N$   $\gamma$ -ray bursts in them, out of the  $N_{\text{tot}} = 1122$  in the BATSE 3B catalog, where  $N = 0, 1, 2, \dots$ . Then I define the observed counts-in-cells distribution,

$$P_N \equiv C_N/N_{\text{cell}} , \tag{1}$$

which is the probability that a randomly chosen cell of size  $\Omega$  has  $N$  bursts in it. The counts-in-cells distribution contains information about clustering of  $\gamma$ -ray bursts on scales comparable to the angular size  $\theta_{\text{rad}}$  of the cell.

I now define  $Q_N$  to be the counts-in-cells distribution that is expected if  $\gamma$ -ray bursts are cosmological in origin and trace the large-scale structure of luminous matter in the universe. This expected distribution depends on only one unknown parameter, the effective distance  $D$  to  $\gamma$ -ray bursts (which I define below), because the angular clustering pattern of bursts on the sky will depend by projection on this distance.

The *likelihood*  $\mathcal{L}$  measures how likely it is that the observed counts-in-cells distribution  $P_N$  is drawn from the expected distribution  $Q_N$ . Since  $Q_N$  depends on the unknown effective distance  $D$  to  $\gamma$ -ray bursts, the likelihood is really a measure of how likely a given value of  $D$  is. It is given by the multinomial expression

$$\mathcal{L} = \frac{N_{\text{cell}}!}{\prod_N C_N!} \prod_N Q_N^{C_N} . \quad (2)$$

Combining equations (1) and (2), and noting that the factorial term is parameter independent, I obtain

$$\log \mathcal{L} = N_{\text{cell}} \sum_N P_N \log Q_N + \text{constant} . \quad (3)$$

The expected counts-in-cells distribution  $Q_N$  is most easily computed using the generating function  $\mathcal{Q}(\lambda) \equiv \sum_N Q_N \lambda^N$  (Peebles 1980; Balian & Schaeffer 1989) and writing it in terms of the means, over the cell size, of the irreducible  $N$ -point angular correlation functions  $\bar{w}_N$  (White 1979; Balian & Schaeffer 1989):

$$\mathcal{Q}(\lambda) = \exp \left[ \langle N \rangle (\lambda - 1) + \sum_{N=2}^{\infty} \bar{w}_N \frac{\langle N \rangle^N}{N!} (\lambda - 1)^N \right] , \quad (4)$$

where the density  $\langle N \rangle = N_{\text{tot}}/N_{\text{cell}}$  is the mean number of bursts in a cell. Indeed, the Poisson distribution,  $Q_N = \langle N \rangle^N \exp(-\langle N \rangle) / N!$ , is obtained (by comparing powers of  $\lambda$ ) when all correlations  $\bar{w}_N$  are zero (uniform distribution) and only the first term in the sum in equation (4) is included.

Now for cosmological  $\gamma$ -ray bursts, the clustering is expected to be weak (LQ) and can be adequately described by one number,  $\bar{w}_2$ , the mean of the two-point angular correlation function  $w(\theta)$  over the cell:

$$\bar{w}_2 = \frac{1}{\Omega^2} \int_{\Omega} d\Omega_1 d\Omega_2 w(\theta_{12}) . \quad (5)$$

Then only the first two terms in the sum in equation (4) contribute to the generating function, and the counts-in-cells distribution is a convolution of two terms:  $Q_N = \sum_{M=0}^N A_{N-M} B_M$ ,

where the first is a Poisson uncorrelated background of single bursts of density  $\langle N \rangle - \langle N \rangle^2 \bar{w}_2$ , and the second a uniform smattering of correlated pairs, with density  $\langle N \rangle^2 \bar{w}_2 / 2$ , in which

$$A_N = \frac{1}{N!} (\langle N \rangle - \langle N \rangle^2 \bar{w}_2)^N e^{-(\langle N \rangle - \langle N \rangle^2 \bar{w}_2)}, \quad (6)$$

$$B_{2k} = \frac{1}{k!} (\langle N \rangle^2 \bar{w}_2 / 2)^k e^{-\langle N \rangle^2 \bar{w}_2 / 2}.$$

Once the angular correlation function is known,  $\bar{w}_2$  can be computed from equation (5), and the expected counts-in-cells distribution  $Q_N$  can be found from equation (6). The likelihood  $\mathcal{L}$  can then be computed from equation (3), and it is a function of the unknown effective distance  $D$  through the dependence of  $\bar{w}_2$  on  $D$ .

## 2.2. Angular Correlation Function

For simplicity, I assume that  $\Omega_0 = 1$  and  $\Lambda = 0$ , and that the large-scale structure clustering pattern is constant in comoving coordinates. The results (cf. § 3) are, in fact, insensitive to these assumptions because of the small redshifts that are involved.

The Limber equation (Peebles 1980), which relates the angular correlation function  $w(\theta)$  to the spatial one, can be recast (Peacock 1991; LQ) in terms of the dimensionless power spectrum  $\Delta^2(k)$ :

$$w(\theta) = \frac{\int_0^\infty r^4 \phi^2(r) dr \int_0^\infty \pi \Delta^2(k) J_0(kr\theta) dk / k^2}{[\int_0^\infty r^2 \phi(r) dr]^2}. \quad (7)$$

Here the selection function  $\phi(r)$  is the probability that a source at comoving distance  $r$  produces a burst that is in the BATSE catalog, i.e., a burst with apparent flux greater than the limiting flux of the BATSE survey. The selection function is a convolution of a source function, representing the luminosity function and the spatial density of the sources,

with an observer function, representing the efficiency with which BATSE detects bursts of a given flux.

Now the cumulative  $C_{\max}/C_{\min}$  distribution of  $\gamma$ -ray bursts seen by BATSE begins to roll over from a  $-3/2$  power law for bursts fainter than  $C_{\max}/C_{\min} \sim 10$  (Meegan et al. 1992). Since this is many times above threshold, it suggests that BATSE sees most of the source distribution and that this distribution is not spatially homogeneous. To the extent that this is the case, the observer function is unity for nearly all the burst sources, irrespective of brightness, so that BATSE sees nearly all of the burst sources. The selection function  $\phi(r)$ , then, depends only on the luminosity function and the mean space density of the sources.

I define  $D$  as the distance beyond which  $\phi(r)$  drops appreciably; thus,  $D$  is the *effective distance* to the “edge” of the source distribution in the BATSE catalog. One could approximate the selection function to be unity out to a comoving distance  $D$ , and zero beyond, but evolution of the luminosity function and/or the mean space density of the sources will change the form of  $\phi(r)$  from  $\theta(r - D)$ ; more generally, I define the effective distance  $D$  as  $\frac{1}{3}D^3 \equiv \int r^2 \phi(r) dr$  (LQ). Thus,  $D$  is not the distance to the very dimmest burst in the BATSE catalog, but rather the typical distance to most of the dim bursts in the sample.

Equation (7) then becomes, upon rescaling  $y \equiv r/D$ ,

$$w(\theta) = \frac{9\pi}{D} \int_0^1 y^4 dy \int_0^\infty \Delta^2(k) J_0(kyD\theta) dk/k^2, \quad (8)$$

so that the angular correlation function satisfies the well-known scaling relation (Peebles 1980)  $w(\theta) = D^{-1}W(D\theta)$  as a function of  $D$ .

I take the power spectrum that characterizes the large-scale clustering of  $\gamma$ -ray burst sources to be the same as that determined from a redshift survey of radio galaxies (Peacock

and Nicholson 1991):

$$\Delta^2(k) = \frac{0.129 (k/k_0)^4}{1 + (k/k_0)^{2.4}}, \quad (9)$$

where  $k_0 = 0.025 h \text{ Mpc}^{-1}$ . This power spectrum is characteristic of moderately rich environments, and is intermediate between that of ordinary galaxies and clusters (LQ). Because the exact bias factor relating the clustering of  $\gamma$ -ray burst sources to that of luminous matter is unknown, such an intermediate *Ansatz* is reasonable. In any case, the resultant distance limit depends only weakly on the bias factor (roughly as the square root).

Substituting equation (9) into equation (8), one obtains the expected intrinsic angular correlation function  $w(\theta)$  of  $\gamma$ -ray bursts as a function of their effective depth  $D$ . Figure 1 shows  $w(\theta)$  (*dashed lines*) for  $D$  of 500, 1000, and 2000  $h^{-1} \text{ Mpc}$ . Note the scaling of  $w(\theta)$  as a function of  $D$ .

Now the observed angular correlation function,  $\tilde{w}(\theta)$ , is smeared at small angular scales because of finite positional errors. Each burst in the BATSE catalog is assigned a positional uncertainty  $\theta_{\text{err}}$  corresponding to a 68% confidence that the true burst position is within an angle  $\theta_{\text{err}}$  to the position listed in the catalog. Defining  $\sigma^2 \equiv (1 - \cos(\theta_{\text{err}})) / 1.14$ , the smearing function can be approximated as a Gaussian of the angular separation  $\Delta\theta$  between the true and observed positions of the burst:  $dP/d\Omega = \exp(-\Delta\theta^2/2\sigma^2)/2\pi\sigma^2$ . If all burst positions are smeared by the same amount,  $\sigma^2$ , the observed angular correlation function is a convolution of the intrinsic correlation function with the smearing function and the modified Bessel function  $I_0$  (Hartmann, Linder, & Blumenthal 1991):

$$\tilde{w}(\theta) = \int \frac{\phi d\phi}{2\sigma^2} w(\phi) e^{-(\theta^2 + \phi^2)/4\sigma^2} I_0\left(\frac{\theta\phi}{2\sigma^2}\right). \quad (10)$$

Figure 1 shows  $\tilde{w}(\theta)$  (*solid lines*) for  $D$  of 500, 1000, and 2000  $h^{-1} \text{ Mpc}$ , with smearing of burst positions of  $\theta_{\text{err}} = 3.8^\circ$ , the median value in the BATSE 3B catalog. Note the smearing of the correlation function on scales smaller than  $\theta_{\text{err}}$ , along with the aliasing



of some small-scale power to angular scales comparable to the positional smearing. It is this smeared correlation function that I use in equation (5) when calculating  $\bar{w}_2$  and the expected counts-in-cells distribution  $Q_N$ , in calculating the likelihood  $\mathcal{L}$  as a function of  $D$ .

The cell size  $\theta_{\text{rad}}$  is chosen in order to maximize the sensitivity of detection, or signal-to-noise ratio, given the strength of the signal expected. The signal  $S$  is given by the total number of correlated pairs in the cells (cf. above eq. [6]):  $S = N_{\text{cell}} \langle N \rangle^2 \bar{w}_2 / 2$ . The noise  $N$  is the square root of the total number of pairs (Peebles 1980; LQ):  $N = (N_{\text{cell}} \langle N \rangle^2 / 2)^{1/2}$ . For a sample of 1122 bursts (corresponding to the total number of bursts in the BATSE 3B catalog) with positional smearing of  $\theta_{\text{err}} = 3.8^\circ$ , the signal-to-noise ratio is maximized when cells of  $\theta_{\text{rad}} = 5^\circ$  are used.

### 3. Results

Figure 2 shows the likelihood of the BATSE 3B catalog data as a function of the effective comoving distance  $D$ , calculated using cells of size  $\theta_{\text{rad}} = 5^\circ$ . The likelihood is normalized to that expected for an isotropic distribution on the sky. At large values of  $D$  (the maximum value allowed is  $D = R_{\text{H}} = 6000 h^{-1}$  Mpc, the size of the horizon in a closed universe), the likelihood goes to unity, because by projection a sufficiently large distance will always lead to an isotropic distribution on the sky. Note also that there is no value of  $D$  for which the likelihood is greater than 1; thus, the maximum likelihood value for  $D$  is  $R_{\text{H}}$ , and the 3B data are consistent with isotropy.

The solid line in Figure 2 shows the likelihood for a positional smearing of  $\theta_{\text{err}} = 3.8^\circ$ , corresponding to the median value in the 3B catalog. To illustrate the dependence of these results on positional errors, I also show (*dashed line*) the results for a larger positional

smearing<sup>3</sup> of  $\theta_{\text{err}} = 6.6$  (with cells of size  $\theta_{\text{rad}} = 9^\circ$  to maximize the signal-to-noise ratio).

Small values of the effective comoving distance to  $\gamma$ -ray bursts are unlikely, according to Figure 2; I find, at the 95% confidence level, that for the 3B median positional error of  $\theta_{\text{err}} = 3.8$ ,  $D$  must be greater than  $630 h^{-1}$  Mpc, corresponding to a redshift  $z > 0.25$ . If the positional errors are larger than quoted and are better characterized by  $\theta_{\text{err}} = 6.6$ , these results are only slightly weakened; at the 95% confidence level,  $D$  must be greater than  $500 h^{-1}$  Mpc, corresponding to a redshift  $z > 0.19$ .

These limits are not sensitive to earlier assumptions (§ 2.2) on cosmology and clustering evolution, since these only become important at higher redshifts. They are also conservative limits, in that a constant median value for the positional errors was used rather than the entire distribution of errors. This is because the bright bursts, which ostensibly are nearer to us, are more clustered (by the scaling property of the correlation function, eq. [8]) and are responsible for the bulk of the expected signal, but, in fact, have smaller errors than the median value. The faint bursts, which are far away, are hardly clustered to begin with (even before smearing), but have errors larger than the median value. Hence, the expected clustering pattern has been smeared more by using a constant median value (this permits a simpler calculation and allows eq. [10] to be used) than by smearing using the entire distribution of errors. Therefore, the counts-in-cells statistic has been weakened somewhat, and thus the quoted lower limits are, in fact, conservative.

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<sup>3</sup>Graziani & Lamb (1996) compare the 3B positions with those from the IPN network and conclude that the systematic errors are larger than the  $1.6$  value quoted in the 3B catalog. Their best-fit model gives a median positional error of  $6.6$ .

#### 4. Discussion

If  $\gamma$ -ray bursts are cosmological and trace the large-scale structure of luminous matter in the universe, and their positional errors are as quoted in the 3B catalog, then the lack of any angular clustering in the data implies that the observed distance to the “edge” of the burst distribution must be farther than  $630 h^{-1}$  Mpc. Since there are 1122 bursts in the catalog, an effective limit on the *nearest* burst to us can be placed by convoluting the likelihood as a function of  $D$  (Fig. 2) with the nearest neighbor distribution of 1122 bursts inside a sphere of radius  $D$ . I find that the nearest burst must be farther than  $40 h^{-1}$  Mpc at the 95% confidence level, and farther than  $10 h^{-1}$  Mpc at the 99.9% level. At this level of confidence, then, none of the bursts that have been observed by BATSE are in nearby galaxies. A signature from the Coma cluster or the “Great Wall” ( $\sim 70 h^{-1}$  Mpc) is not likely to be seen in the data at present, since only a few bursts could have originated from these distances. Indeed, a search for such a signature (Hartmann, Briggs, & Mannheim 1996) found no compelling evidence for anisotropy in supergalactic coordinates.

The median distance to the nearest burst is  $170 h^{-1}$  Mpc. Since the brightest burst in the 3B catalog has a fluence of  $7.8 \times 10^{-4}$  ergs  $\text{cm}^{-2}$  in  $\gamma$ -rays, this implies that the total energy released in  $\gamma$ -rays during a burst event is of order  $3 \times 10^{51} h^{-2}$  ergs.

As the number of observed  $\gamma$ -ray bursts keeps increasing, the distance limit will improve. In fact, LQ showed that, with 3000 burst locations, the clustering of bursts might just be detectable and would provide compelling evidence for a cosmological origin. If it is not detected, the redshift to the “edge” of the bursts would be put at  $z \sim 1$  or beyond.

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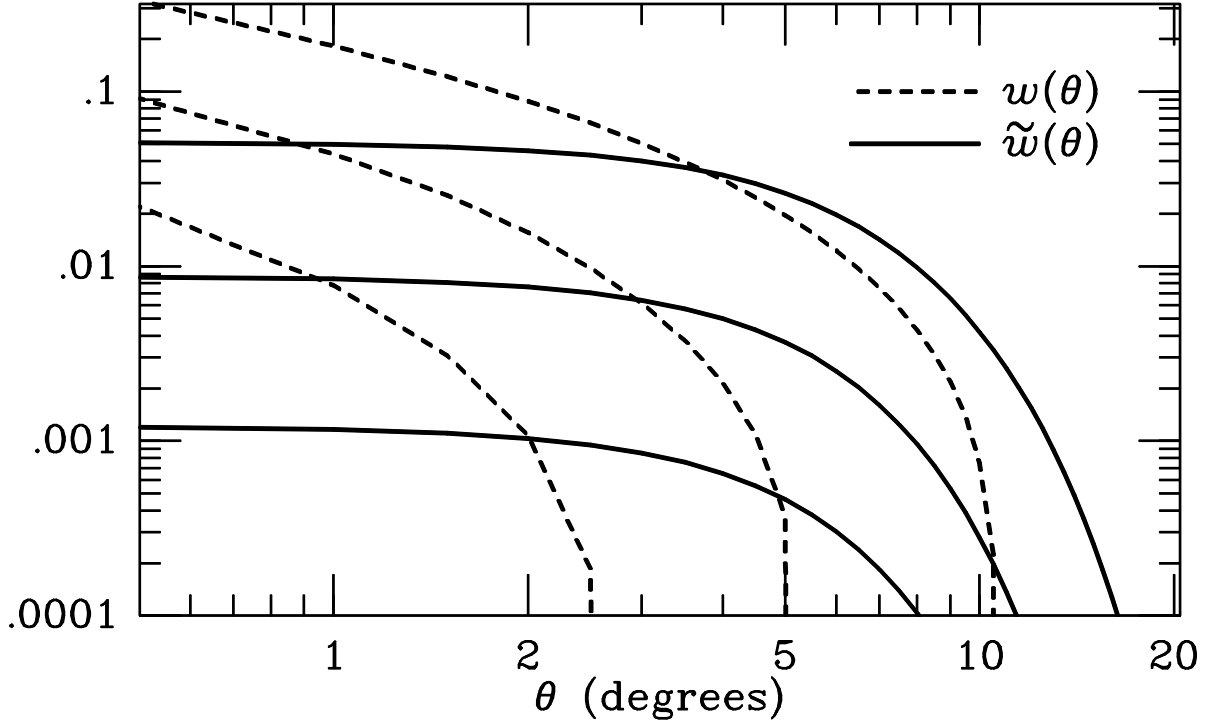


Fig. 1.— Angular correlation function and its dependence on the effective comoving distance  $D$  to  $\gamma$ -ray bursts. Shown are both the intrinsic,  $w(\theta)$  (*dashed line*), and smeared,  $\tilde{w}(\theta)$  (*solid line*), correlation functions, in decreasing amplitude, for  $D = 500, 1000$ , and  $2000 h^{-1}$  Mpc. The smearing corresponds to  $\theta_{\text{err}} = 3.8$ , the median value in the BATSE 3B catalog (see text).

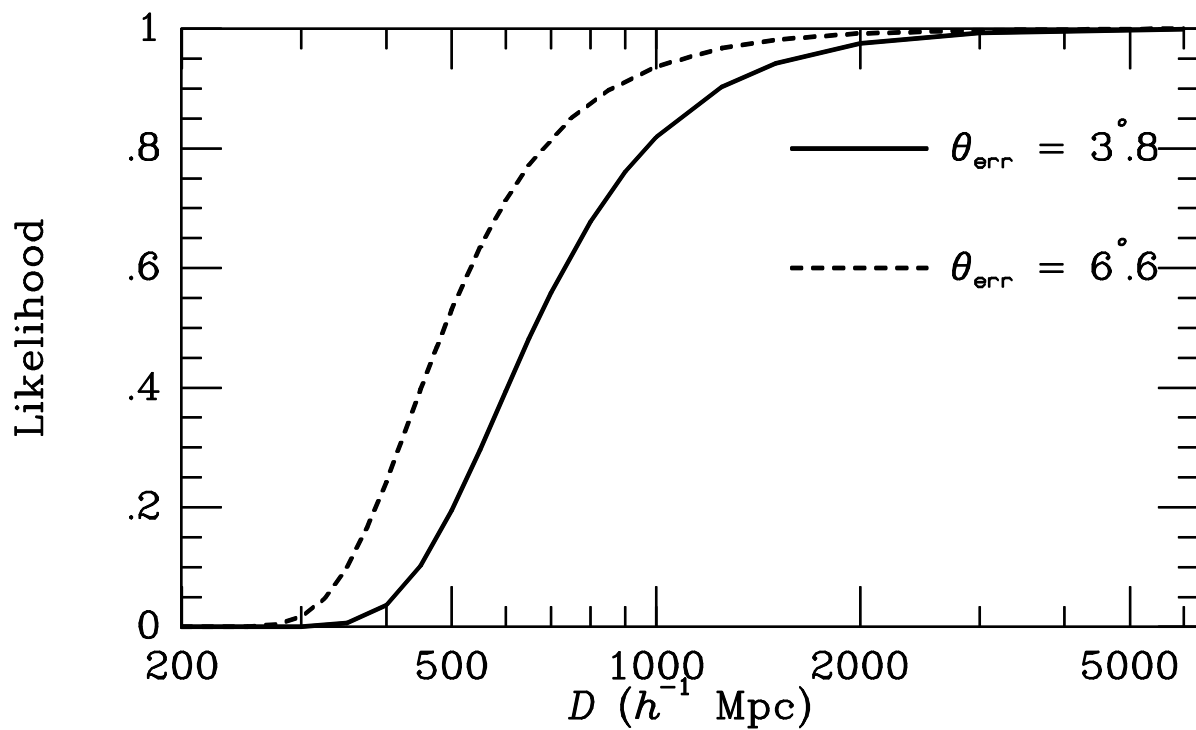


Fig. 2.— Likelihood of the BATSE 3B catalog data as a function of the effective comoving distance  $D$  to  $\gamma$ -ray bursts, shown with a smearing of  $\theta_{\text{err}} = 3.8^\circ$  and  $6.6^\circ$  (see text).